Principles of Energy Conversion

Part 3. Exponential Growth & Hubbert's Peak

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Article 4

Exponential Growth Rate

Development of energy policies by governments and technology decisions by companies center on the consequences of growth of some energy parameter. These parameters may be energy consumption such as residential electric power, an energy source such as wind, or use of energy reserves such as coal. Growth is typically considered on an annual basis with a constant fractional increase in the parameter of interest. The concept discussed herein is equally useful for studies of population growth, gross national product, cost of living, radioisotope half-life, and solar energy attenuation.

Consider some quantity P which increases at a constant percentage in time.

$P \equiv$ quantity (power, energy supply, etc.)

$$i = \frac{\% \text{ of } P}{\text{time}}$$

The time rate of change of P is equal to a constant fractional rate of P:

$$\frac{dP}{dt} = iP$$

The increase in P after some time can be found by integrating dP/dt from time t_0 to t, where t_0 is the initial time for which the quantity is P_0.

$$\ln\left(\frac{P}{P_0}\right) = i(t - t_0)$$

For convenience, t_0 is considered equal to zero. Subsequently, the rate of growth of quantity P is exponential in time.

$$P = P_0 e^{it}$$

4.1 Doubling Time

An important concept associated with exponential growth is doubling time, t_d; that is, the time period required to double P.

$$\frac{P}{P_0} = 2 \quad \longrightarrow \quad t_d = \frac{\ln 2}{i} = \frac{0.693}{i}$$

Example:

From 1920 to 1975, the average annual growth rate of electrical energy was 7% per year:

$$i = \frac{0.07}{\text{year}}$$

This meant that the capacity for electrical power generation had to double every 9.9 years in order to keep up with demand.

$$t_d = \frac{0.693}{0.07/\text{year}} = 9.9 \text{ years}$$

4.1.1 Normalization With Respect to Doubling Time

The exponential growth rate can be conveniently expressed by normalizing time with the doubling time as shown in Figure 4.1.

$$\ln\left(\frac{P}{P_0}\right) = 2 = i\left(\frac{t}{t_d}\right)t_d$$

Substituting for $t_d = \ln 2/i$,

$$\frac{P}{P_0} = 2^{t/t_d}$$

4.2 Cumulative Growth

The total accumulation of P over a time period is found by integrating P with respect to time. If E represents the cumulative P, then

$$E = \int_{t_1}^{t_2} P\,dt = \int_{t_1}^{t_2} P_0 e^{it}\,dt = \frac{1}{i}P_0\left[e^{it_2} - e^{it_1}\right]$$

which can be rearranged to the following form:

$$E = \frac{1}{i}P_0 e^{it_1}\left[e^{i(t_2-t_1)} - 1\right]$$

For convenience, we define E_0 as the total accumulation of P from time $t = -\infty$ to $t = t_1$.

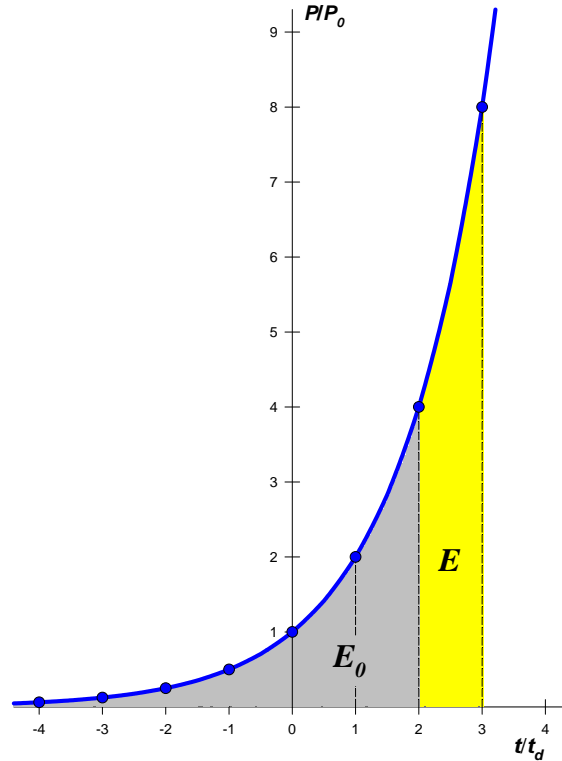$$E_0 = \int_{-\infty}^{t_1} P_0 e^{it}\,dt = \frac{1}{i}P_0\left[e^{it_1} - 0\right] = \frac{1}{i}P_0 e^{it_1}$$

Figure 4.1: Cumulative growth. The total quantity E during one doubling period is equal to the total for all previous time.

If $t_2 - t_1$ is equal to a doubling period, t_d, then $e^{it_d} = 2$, which results in $E = E_0$.

$$E = \frac{1}{i} P_0 e^{it_1} \left[2 - 1 \right] = E_0$$

The implication is that the total accumulation of P during one doubling period is equal to the total accumulation of P for all time prior to the doubling period!

4.3 Examples of Exponential Growth in Energy Utilization

4.3.1 Example: Growth in U.S. Energy Consumption

Using the annual energy consumption in the U.S. for the year 1970 and using a projection of 4% annual growth in energy consumption, determine the doubling time and the energy consumption for the year 2000.

The 1970 US annual energy consumption was 67.5 Quad[1] which is equivalent to $71.2 \cdot 10^{18}$ J.[2] For a 4%/year growth rate, the doubling time is:

$$t_d = \frac{\ln 2}{0.04/\text{year}} = 17.325 \text{ years}$$

Power consumption in the year 2000 is estimated to be:

$$P_{30} = P_0 e^{it_{30}} = (71.2 \cdot 10^{18} \text{ J}) e^{(0.04/\text{year})(30 \text{ years})} = 235.6 \cdot 10^{18} \text{ J}$$

Alternatively,

$$P_{30} = P_0 2^{(t/t_d)} = (71.2 \cdot 10^{18} \text{ J}) 2^{(30 \text{ years}/17.375 \text{ years})} = 235.6 \cdot 10^{18} \text{ J}$$

The actual U.S. energy consumption in 2000 was 98.2 Quad.[1] The estimated growth rate of 4% per year was off by a factor of 2.274. The actual *average* annual growth rate in US energy consumption for the 30 year period between 1970 and 2000 is:

$$\frac{P_{2000}}{P_{1970}} = e^{it} \quad \longrightarrow \quad i = \frac{1}{t} \ln\left(\frac{P_{2000}}{P_{1970}}\right) = \frac{1}{30 \text{ years}} \ln\left(\frac{98.2 \text{ Quad}}{67.5 \text{ Quad}}\right) = 1.25\%$$

4.3.2 Example: Additional Coal Reserve Utilization

If the demonstrated reserves of coal increased by an order of magnitude due to a combination of new discoveries and increased return on investments, how long would the coal supply all of the U.S. energy consumption assuming 100% conversion efficiency?

$$t = \frac{1}{(0.0125 \text{ yr})} \ln\left(\frac{\left[(105 \cdot 10^{21} \text{ J})(0.0125/\text{year})\right]}{107.2 \cdot 10^{18} \text{ J/year}} + 1\right) = 207 \text{ years}$$

At a growth rate of 1.25%/year, a factor of 10 increase in coal reserves does not change the time the coal will last from 64 years to 640 years. Only an additional 143 years is obtained unless the growth rate is reduced.

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[1]from Energy Information Agency [1]
[2]1 Btu $\equiv$ 1055 J

4.3.3 Example: Utilization of Coal Reserves

Estimate how long the demonstrated reserves of U.S. coal would supply *all* of the US energy consumption at an annual growth rate of 1.25%. Assume a conversion efficiency of 100% for the coal.

The total demonstrated reserves of coal in the U.S. as of 2007 is 491.1 billion short tons.[3] One short ton of coal is considered to be equivalent to 20.3 million Btu.[1] The total estimated energy value in the demonstrated reserves of U.S. coal is:

$$\left(491.1 \cdot 10^9 \text{ short tons} \right) \left(\frac{20.3 \cdot 10^6 \text{ Btu}}{\text{short ton of coal } [\eta = 1]} \right) = 9.97 \cdot 10^{18} \text{ Btu} = 10.5 \cdot 10^{21} \text{ J}$$

The total energy consumption in the U.S. for the year 2007 is $101.6 \cdot 10^{15}$ Btu $= 107.2 \cdot 10^{18}$ J.[1]

$$E = E_0 \left[e^{it} - 1 \right] \quad \longrightarrow \quad t = \frac{1}{i} \ln \left(\frac{E}{E_0} + 1 \right) = \frac{1}{i} \ln \left(\frac{Ei}{P_0} + 1 \right)$$

$$t = \frac{1}{(0.0125 \text{ year})} \ln \left[\frac{\left(10.5 \cdot 10^{21} \text{ J} \right) \left(0.0125/\text{year} \right)}{107.2 \cdot 10^{18} \text{ J/year}} + 1 \right] = 64 \text{ years}$$

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[3]from Energy Information Agency [1]

4.4 Review of Exponentials and Logarithms

4.4.1 Rules for Exponentials

$$a^m \times a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$a^{-t} = \frac{1}{a^t} \qquad a^0 = 1$$

$$a^{p/q} = \sqrt[q]{a^p}$$

4.4.2 Rules for Logarithms

If $x = a^y$, then $y = \log_a x$ where a is the 'base' of the logarithm. The two most commonly used bases are base 10 and base e.

$$\log_a(xy) = \log_a x + \log_a y \qquad \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^n) = n\log_a x$$

$$\log_b a = \frac{1}{\log_a b} \qquad \log_a x = \frac{\log_b x}{\log_b a} = \log_b x \times \log_a b$$

4.4.3 Common vs Natural Logarithms

base 10: 'common' or Briggsian logarithms. Abbreviated as $\log$.

$$\log_{10} x \equiv \log x$$

base e: 'natural', Napernian, or hyperbolic logarithms. Abbreviated as $\ln$.

$$\log_e x \equiv \ln x \ ; \quad e = 2.71828\ldots$$

relation between base e and base 10:

$$\ln x = \frac{\log_{10} x}{\log_{10} e} = \log_e 10 \times \log_{10} x$$

$$\log_e 10 = \ln 10 = 2.30258\,50930$$

Therefore, $\ln x = 2.3026 \log x$.

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Hubbert's Peak

Figure 5.1: M. King Hubbert [2]

M. King Hubbert (1903–1989) was born in central Texas in 1903 where he attended one- and two-teacher country schools until the ninth grade and then went for a year to a small private school, from which he graduated in 1921. He spent the next two years at Weatherford College before leaving to work in the wheat fields of Oklahoma and Kansas and as a gandy dancer on the Union Pacific Railroad, en route to the University of Chicago. In Chicago, while attending university, he was successively a telephone installer, a postal clerk, a waiter, a checkroom attendant, a supernumerary in *Romeo and Juliet* and in various operas and a camp cook in Wisconsin. Hubbert received his bachelors, masters, and doctorate degrees in geology and geophysics from Columbia University. He worked for the Shell Oil Company for 20 years before joining the the U.S. Geological Survey in 1964. [3]

There has always been interest in understanding the depletion rate of an initially fixed resource; at the local level (coal mine), field level (Mesabi Iron Range), national level and global level. The initial stage of production or extraction has historically been observed to increase exponentially. This begs the question "How many doubling periods can be sustained before production rates will reach astronomical magnitudes?" [4].

Coal mine production and early oil field data from Ohio and Illinois exhibited this exponential growth, a leveling off of production and subsequent decline in production. Hubbert used this historical data to attempt to predict when U.S. crude oil production would peak even though at the time of his analysis crude oil production was only at 50% capacity. His goal was to predict when production would decline. He used growth in production to predict consumption.

A semilog plot of production data allowed for determination of the growth rate, demonstrated in Figure 5.2b. Hubbert had excellent estimates of the ultimate reserves of U.S. petroleum. Knowing the exponential growth rate and the total cumulative, he predicted that peak oil production in U.S. crude oil would occur around 1968. He used two different estimates for the ultimate reserves as shown in Figure 5.3.

When Hubbert made his prediction in 1956 that the peak of production would occur in only 10 years, the U.S. producers were only at 50% of production capacity. His

(a) Linear scale

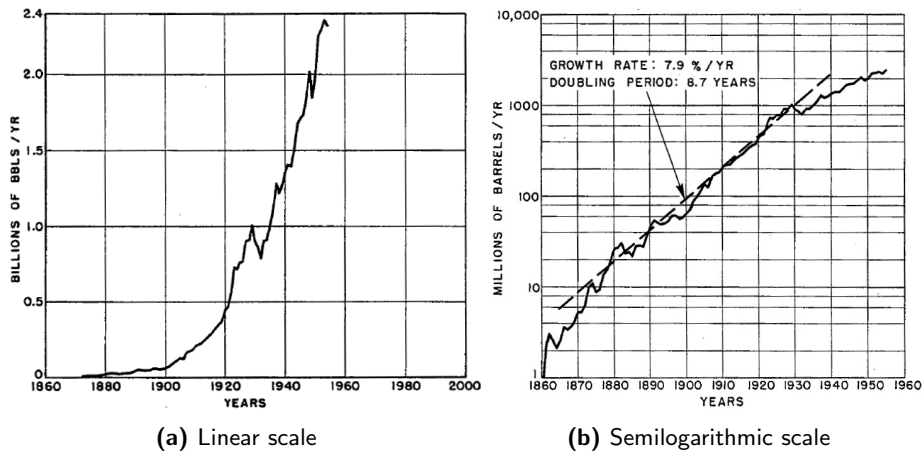**(b)** Semilogarithmic scale

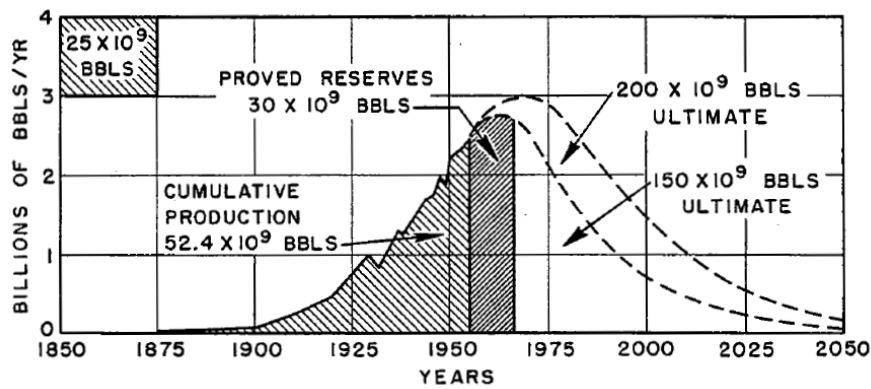**Figure 5.2:** U.S. crude oil production rate from 1860 to 1956. [4]

Figure 5.3: Ultimate United States crude-oil production based on assumed initial reserves of 150 and 200 billion barrels. [4]

prediction was not seriously considered by the industry nor by the government. U.S. production of oil peaked in 1972, but likely would have peaked in 1968 as predicted by Hubbert had not the U.S. begun importation of oil. [5]

5.1 Why Hubbert's Model Worked So Well

Hubbert's model worked well and it has been difficult to repeat his success with later predictions. Hubbert had excellent estimates for the demonstrated reserve proved plus indicated) of U.S. petroleum. More importantly, prices on energy were kept artificially low (see 5.1.1 below) while maintaining healthy profits for producers. This resulted in an unrestrained exponential growth in petroleum consumption [5]. The supply of

oil appeared to be endless because there was no significant fluctuation in energy costs that might indicate resource exhaustion. As a result, energy markets were extremely stable both economically and politically. Even after U.S. wells had reached maximum production, the availability of imports offset the production limitations of the U.S. producers that maintained the perception of a limitless resource even after U.S. wells are reached maximum production capacity.

5.1.1 Texas Railroad Commission

The artificially constrained petroleum price was a federal policy intended to provide stability for the country. In the 1870s there were dire warnings that the U.S. would run out of petroleum. The discovery of the East Texas fields completely changed this viewpoint. There was suddenly an oversupply of oil and a dramatic collapse in prices at the onset of the Great Depression. From 1929 to 1932, demand dropped by 22% and the price of crude dropped from \$1.15/bbl to \$0.10/bbl[1] [5]. The U.S. and state governments instituted proration (production limits) in order to stabilize the industry. The result was an increase in prices that was sufficiently low to be acceptable by consumers, yet high enough for producers to make a profit. Efficient producers could make twenty-five times the production cost at these prices. The responsibility for regulating well-head production was given to the Texas Railroad Commission. This governing body capped production on wells in order to maintain uniform supply at a generally constant price. OPEC[2] is modeled after the Texas Railroad Commission. Figure 5.4 compares the annual production to the price per barrel (2008 dollars).
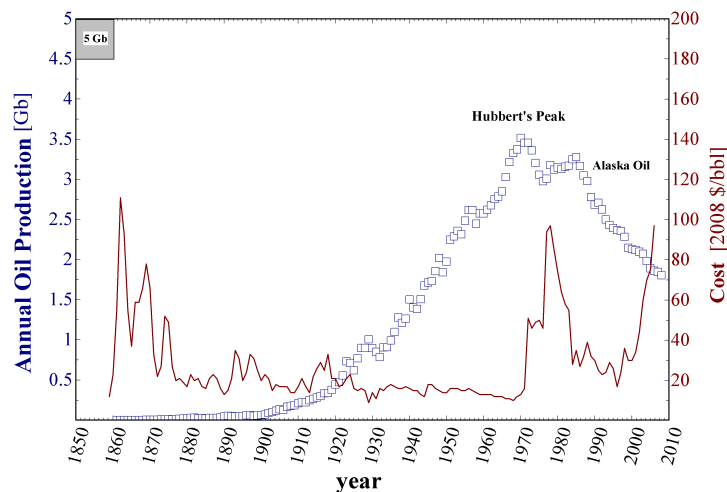

Figure 5.4: Historical U.S. crude oil production from 1860 to 2006. The price per barrel in 2008 dollars shown as solid line. Data obtained from the U.S. DOE Energy Information Agency (EIA).

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[1] in 1930's dollars
[2] The Organization of the Petroleum Exporting Countries

5.2 Peak Production Prediction and the Effect of New Reserves

There is much debate about how discovery of new reserves affect the prediction of 'peak oil'. The plot shown in Figure 5.5 illustrates how little new reserves extend the decrease in production. Each square represents 1 billion quantities of a resource. An example would be barrels of crude. If the known reserves are 20 Gb with a moderate growth rate, the resource would last approximately 90 years. If at year 20, a second reserve is discovered that is approximately half the size of the original, there is negligible increase in the life of the resource. If a third reserve at 6 Gb is discovered at year 80, the increase in resource life is only another 20 years.
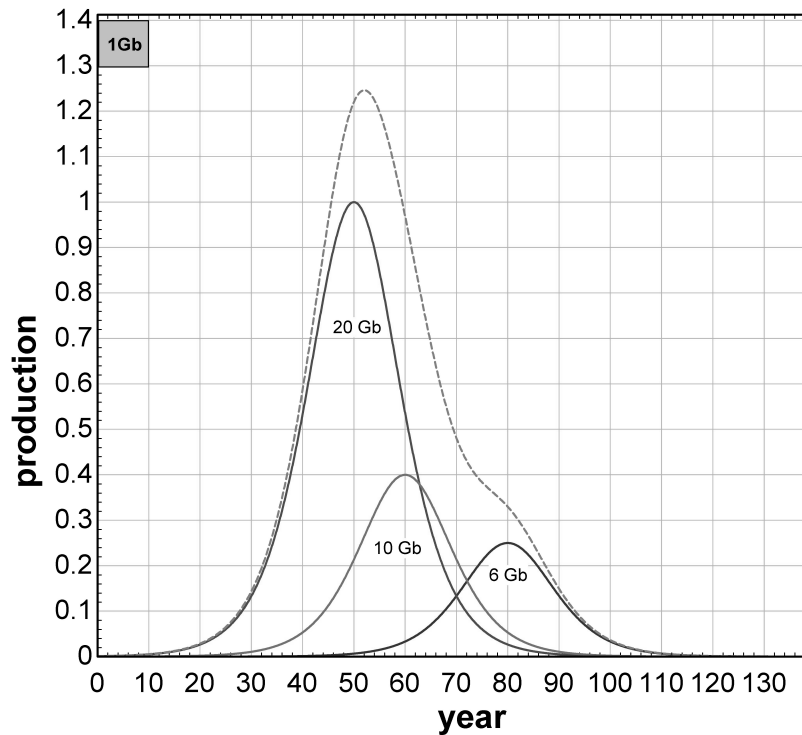

Figure 5.5: Illustration on the effect of new reserves on cumulative production.

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[1] Energy Information Agency. Annual Energy Review 2008. Technical report, U.S. Department of Energy, June 2009.

[2] Kenneth S. Deffeyes. *Hubbert's Peak: The Impending World Oil Shortage*. Princeton University Press, 2001.

[3] M. K. Hubbert. The Energy Resources of the Earth. *Scientific American*, 225, No. 3 (September 1971), pp. 60-73(3):60–73, September 1971.

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[5] Alfred J. Cavallo. Hubbert's Petroleum Production Model: An Evaluation and Implications for World Oil Production Forecasts. *Natural Resources Research*, 13(4):211–221, December 2004.